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Mark Scheme (Results)
January 2008

GCE

GCE Mathematics (6684/01)

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Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1. (a)	A census is when every member of the population is investigated.	B1
(b)	There would be no cookers left to sell.	B1
(c)	A list of the unique identification numbers of the cookers.	B1
(d)	A cooker	B1
		(4)
Notes		
1. (a)	**B1** Need one word from each group (1) Every member /all items / entire /oe (2) population/collection of individuals/sampling frame/oe	
	enumerating the population on its own gets B0	
(b)	**B1** Idea of Tests to destruction. Do not accept cheap or quick	
(c)	**B1** Idea of list/ register/database of cookers/serial numbers	
(d)	**B1** cooker(s) / serial number(s)	
	The sample of 5 cookers or every 400^{th} cooker gets B1	

2 (a)	Let X be the random variable the number of faulty bolts	M1
	$P(X \le 2) - P(X \le 1) = 0.0355 - 0.0076$ or $(0.3)^2(0.7)^{18}\frac{20!}{18!2!}$	A1
	$= 0.0279$ $= 0.0278$	(2)
(b)	$1 - P(X \le 3) = 1 - 0.1071$	M1
	$= 0.8929$	A1
		(2)
	or $1-(0.3)^3(0.7)^{17}\frac{20!}{17!3!}-(0.3)^2(0.7)^{18}\frac{20!}{18!2!}-(0.3)(0.7.)^{19}\frac{20!}{19!1!}-(0.7)^{20}$	
(c)	$\frac{10!}{4!6!}(0.8929)^6(0.1071)^4 = 0.0140.$	M1A1√A1
		(3)
Notes:		
2. (a)	**M1** Either attempting to use P ($X \le 2$) – P ($X \le 1$) or attempt to use binomial and find p(X = 2). Must have $(p)^2(1-p)^{18}\frac{20!}{18!2!}$, with a value of p **A1** awrt 0.0278 or 0.0279.	
(b)	**M1** Attempting to find 1 – P($X \le 3$) **A1** awrt 0.893	
(c)	**M1** for $k\ (p)^6(1-p)^4$. They may use any value for p and k can be any number or $^nC_6p^6(1-p)^{n-6}$ **A1**√ $\frac{10!}{4!6!}(their\ part\ b)^6(1-their\ part\ b)^4$ may write $^{10}C_6$or $^{10}C_4$ **A1** awrt 0.014	
		B1 B1
		(2)

3. (a)	<u>Events</u> occur at a constant rate. any two of the 3 <u>Events</u> occur independently or randomly. <u>Events</u> occur singly.	B1 M1
(b)	Let X be the random variable the number of cars passing the observation point.	A1
(i)	Po(6) $P(X \le 4) - P(X \le 3) = 0.2851 - 0.1512$ or $\frac{e^{-6}6^4}{4!}$ $= 0.1339$	M1 A1 (5)
(ii)	$1 - P(X \le 4) = 1 - 0.2851$ or $1 - e^{-6}\left(\frac{6^4}{4!}+\frac{6^3}{3!}+\frac{6^2}{2!}+\frac{6}{1!}+1\right)$ $= 0.7149$	B1 M1 A1 A1 (4)
(c)	P (0 car and 1 others) + P (1 cars and 0 other) $= e^{-1} \text{ x } 2e^{-2} + 1e^{-1} \text{ x } e^{-2}$ = 0.3679 x 0.2707 + 0.3674 x 0.1353 = 0.0996 + 0.0498 = 0.149 <u>alternative</u> $P_o(1+2) = P_o(3)$ B1 $P(X=1) = 3e^{-3}$ M1 A1 = 0.149 A1	
Notes 3(a)	**B1 B1** Need the word events at least once. Independently and randomly are the same reason. Award the first B1 if they only gain 1 mark **Special case.** If they have 2 of the 3 lines without the word events they get B0 B1 **B1** Using Po(6) in (i) or (ii)	
(b) (i)	**M1** Attempting to find $P(X \le 4) - P(X \le 3)$ or $\frac{e^{-\lambda}\lambda^4}{4!}$	

(ii)	**A1** awrt 0.134 **M1** Attempting to find $1 - P(X \le 4)$ **A1** awrt 0.715	
(c)	**B1** Attempting to find both possibilities. May be implied by doing $e^{-\lambda_1} \times \lambda_2 e^{-\lambda_2} + e^{-\lambda_2} \times \lambda_1 e^{-\lambda_1}$ any values of λ_1 and λ_2 **M1** finding one pair of form $e^{-\lambda_1} \times \lambda_2 e^{-\lambda_2}$ any values of λ_1 and λ_2 **A1** one pair correct **A1** awrt 0.149 Alternative. B1 for Po(3) M1 for attempting to find P(X=1) with Po(3) A1 $3e^{-3}$ A1 awrt 0.149	

4. (a)	$K(2^4 + 2^2 - 2) = 1$	M1
	$K = 1/18$	A1
		(2)
(b)	$1 - F(1.5) = 1 - \frac{1}{18}(1.5^4 + 1.5^2 - 2)$	M1
	$= 0.705$ or $\frac{203}{288}$	A1
		(2)
(c)	$f(y) = \begin{cases} \frac{1}{9}(2y^3 + y) & 1 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$	M1 A1 B1
		(3)
Notes		
4. (a)	**M1** putting F(*2*) = 1 or F(*2*) – F(1) = 1 **A1** cso. Must show substituting $y = 2$ and the 1/18	
(b)	**M1** either attempting to find 1 – F(1.5) may write and use F(2) – F (1.5) **A1** awrt 0.705	
(c)	**M1** attempting to differentiate. Must see either a $y^n \to y^{n-1}$ at least once **A1** for getting $\frac{1}{9}(2y^3 + y)$ o.e and $1 \le y \le 2$ allow $1 < y < 2$ **B1** for the 0 *otherwise.* Allow 0 for y <1 and 0 for y >2 Allow them to use any letter	

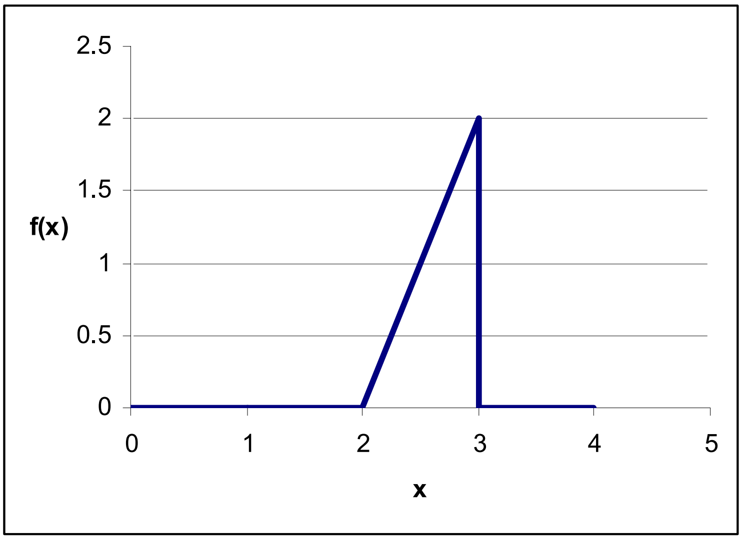
5	$H_0 : p = 0.3$; $H_1 : p > 0.3$ Let X represent the number of tomatoes greater than 4 cm : $X \sim B(40, 0.3)$ $P(X \geq 18) = 1 - P(X \leq 17)$ $= 0.0320$ $P(X \geq 18)\ 1 - P(X \leq 17) = 0.0320$ $P(X \geq 17) = 1 - P(X \leq 16) = 0.0633$ CR $X \geq 18$ $0.0320 < 0.05$ $18 \geq 18$ or 18 in the critical region no evidence to Reject H_0 or it is significant New fertiliser has <u>increased</u> the probability of a <u>tomato</u> being greater than 4 cm **Or** Dhriti's claim is true	B1 B1 B1 M1 A1 M1 B1d cao (7)
5	**B1** for correct H_0 . must use p or pi **B1** for correct H_1 must use p and be one tail. **B1** using B(40, 0.3). This may be implied by their calculation **M1** attempt to find $1 - P(X \leq 17)$ or get a correct probability. For CR method must attempt to find $P(X \geq 18)$ or give the correct critical region **A1** awrt 0.032 or correct CR. **M1** correct statement based on their probability , H_1 and 0.05 or a correct contextualised statement that implies that. **B1** this is not a follow through .conclusion in context. Must use the words increased, tomato and some reference to size or diameter. This is dependent on them getting the previous M1 If they do a <u>**two tail test**</u> they may get B1 B0 B1 M1 A1 M1 B0 For the second M1 they must have accept Ho or it is not significant or a correct contextualised statement that implies that. .	

6a (i)	Let X represent the number of sunflower plants more than 1.5m high	
	$X \sim \text{Po}(10)$ $\mu=10$	
	$P(8 \le X \le 13) = P(X \le 13) - P(X \le 7)$	
	$= 0.8645 - 0.2202$	B1
	$= 0.6443$ awrt 0.644	M1
ii)	$X \sim N(10,7.5)$	
	$P(7.5 \le X \le 13.5) = P\left(\frac{7.5-10}{\sqrt{7.5}} \le X \le \frac{13.5-10}{\sqrt{7.5}}\right)$	A1 B1
	$= P(-0.913 \le X \le 1.278)$	M1 M1
	$= 0.8997 - (1 - 0.8186)$	
	$= 0.7183$ awrt 0.718 or 0.719	A1 A1
b)	Normal approx /not Poisson since (n is large) and p close to half. **or** (np = 10 npq = 7.5) mean $\neq$ variance **or** np (= 10) and nq (= 30) both >5. or exact binomial = 0.7148	M1 A1 (10) B1 B1dep (2)
6a (i)	**B1** mean = 10 May be implied in (i) or (ii) **M1** Attempting to find $P(X \le 13) - P(X \le 7)$ **A1** awrt 0.644	
ii)	**B1** $\sigma^2 = 7.5$ May be implied by being correct in standardised formula **M1** using 7.5 or 8.5 or 12.5 or 13.5. **M1** standardising using 7.5 or 8 or 8.5 **or** 12.5 or13 or 13.5 and their mean and standard deviation.	

	A1 award for either $\frac{7.5-10}{\sqrt{7.5}}$ or awrt -0.91 **A1** award for either $\frac{13.5-10}{\sqrt{7.5}}$ or awrt 1.28 **M1** Finding the correct area. Following on from their 7.5 and 13.5. Need to do a Prob >0.5 – prob <0.5 or prob <0.5 + prob< 0.5 **A1** awrt 0.718 or 0.719 only. Dependent on them getting all three method marks. No working but correct answer will gain all the marks		
b)	first **B1** normal second **B1** p close to half, or mean $\neq$ variance or np and nq both > 5.They may use a number bigger than 5 or they may work out the exact value 0.7148 using the binomial distribution. **Do not allow np> 5 and npq>5**		

7 ai)	A hypothesis test is a mathematical procedure to examine a value of a population parameter proposed by the null hypothesis compared with an alternative hypothesis.	B1
ii)	The critical region is the range of values **or** a test statistic or region where the test is significant that would lead to the rejection of H_0.	B1g B1h (3)
(b)	Let X represent the number of incoming calls : $X \sim Po(9)$ From table $P(X \geq 16) = 0.0220$ $P(x \leq 3) = 0.0212$ Critical region ($x \leq 3$ or $x \geq 16$)	B1 M1 A1 A1 B1 (5)
(c)	Significance level = 0.0220 + 0.0212 = 0.0432 or4.32%	B1 (1)
(d)	$H_o : \lambda = 0.45$; $H_1 : \lambda < 0.45$ (accept : $H_o : \lambda = 4.5$; $H_1 : \lambda < 4.5$) Using $X \sim Po(4.5)$ $P(X \leq 1) = 0.0611$ CR $X \leq 0$ awrt 0.0611 $0.0611 > 0.05$. $1 \geq 0$ or 1not in the critical region There is evidence to Accept H_0 or it is not significant There is no evidence that there are less calls during school holidays.	B1 M1 A1 M1 B1cao (5)
Notes 7 ai)	B1 Method for deciding between 2 hypothesis.	
ii)	B1 range of values. This may be implied by other words. Not region on its own B1 which lead you to reject H_0	

(b) (c) (d)	Give the first B1 if only one mark awarded. B1 using $P_o(9)$ M1 attempting to find $P(X \geq 16)$ or $P(x \leq 3)$ A1 0.0220 or $P(X \geq 16)$ A1 0.0212 or $P(x \leq 3)$ These 3 marks may be gained by seeing the numbers in part c B1 correct critical region A completely correct critical region will get all 5 marks. Half of the correct critical region eg $x \leq 3$ or $x \geq 17$ say would get B1 M1 A0 A1 B0 if the M1 A1 A1 not already awarded. B1 cao awrt 0.0432 B1 may use λ or μ. Needs both H_0 and H_1 M1 using $P_o(4.5)$ A1 correct probability or CR only M1 correct statement based on their probability , H_1 and 0.05 or a correct contextualised statement that implies that. **B1** this is not a follow through .Conclusion in context. Must see the word **calls** in conclusion If they get the correct CR with no evidence of using $P_o(4.5)$ they will get M0 A0 SC If they get the critical region $X \leq 1$ they score M1 for rejecting H_0 and B1 for concluding the rate of calls in the holiday is lower.	

8. a)	 Max height of 2 labelled and goes through(2,0) shape must be between 2 and 3 and no other lines drawn (accept patios drawn) / correct shape 3	B1 B1 B1 (3) B1 (1)
b)	$\int_2^3 2x(x-2)\,dx = \left[\frac{2x^3}{3} - 2x^2\right]_2^3$	M1A1
c)	$= 2\frac{2}{3}$	A1 (3)
	$\int_2^m 2(x-2)\,dx = 0.5$	
d)	$[x^2 - 4x]_2^m = 0.5$ $m^2 - 4m + 4 = 0.5$ $m^2 - 4m + 3.5 = 0$ $m = \frac{4 \pm \sqrt{2}}{2}$ $m = 2.71$	M1 A1 M1 A1 (4)
e)	Negative skew. mean < median < mode .	B1 B1dep (2)

Notes 8.	**B1** the graph must have a maximum of 2 which must be labelled	
(a)	**B1** the line must be between 2 and 3 with not other line drawn except patios. They can get this mark even if the patio cannot be seen.	
	B1 the line must be straight and the right shape.	
(b)	**B1** Only accept 3	
(c)	**M1** attempt to find $\int x\mathrm{f}(x)\mathrm{d}x$ for attempt we need to see $x^{n} \rightarrow x^{n+1}$. ignore limits	
	A1 correct integration ignore limits	
	A1 accept $2\frac{2}{3}$ or awrt 2.67 or $2.\dot{6}$	
(d)	**M1** using $\int \mathrm{f}(x)\mathrm{d}x = 0.5$	
	A1 $m^2 - 4m + 4 = 0.5$ oe	
	M1 attempting to solve quadratic.	
	A1 awrt 2.71 or $\frac{4+\sqrt{2}}{2}$ or $2+\frac{\sqrt{2}}{2}$ oe	
(e)	First **B1** for negative Second **B1** for mean < median< mode. Need all 3 or may explain using diagram.	